Solution to Problem Three, Homework Two

1. Method One: Ratios

This method requires us to use Kepler's third law in a ratio. We have one equation which is for the moon, and another for the satellite. The period and distance for the moon are represented by P_m and R_m. For the satellite we will use P_s and R_s for the same variables. Thus we have,

$$\frac{P_s^2}{P_m^2} = \frac{kR_s^3}{kR_m^3} \tag{1}$$

After cancelling the constant k's, this simplifies to the ratio,

$$(\frac{P_s}{P_m})^2 = (\frac{R_s}{R_m})^3 \tag{2}$$

Rearranging to solve for R_s, we get,

$$R_s = R_m(\frac{P_s}{P_m})^{2/3} \tag{3}$$

By plugging in the appropriate values for the period of the moon and satellite, as well as the radius of the moon, we get a value for the radius of the orbit for a geosynchronous satellite. We will use $P_s = 1$ day, $P_m = 27.3$ days, and $R_m = 3.8x10^{10}$ centimeters.

$$R_s = 3.8x10^{10}(\frac{1}{27.3})^{2/3} = 4.2x10^9 cm \tag{4}$$

2. Method Two: Calculate k

This method is similar to Method One, but splits it into two stages. First, we use the moon equation to solve for the constant k.

$$k = \frac{P_m^2}{R_m^3} = \frac{27.3^2}{(3.8x10^{10})^3} = 1.36x10^{-29} \tag{5}$$

This works for a variety of units, so long as they are kept consistent. Our value for k is then entered into the equation for the satellite, and we solve for R_s,

$$R_s = (\frac{P_s^2}{k})^{1/3} = (\frac{1^2}{1.36x10^{-29}})^{1/3} = 4.2x10^9 cm \tag{6}$$

3. Method Three: Formula

In the previous two methods, we used the information given on the moon's orbit. However, some noticed that if we know the earth's mass, we don't need the moon information, and can instead use Newton's more complete formula,

$$P^2 = \frac{4\pi^2 R^3}{GM_E} \tag{7}$$

This equation, when solved for R, will give us the correct radius only if cgs or SI units are used throughout.

$$R = (\frac{P^2 GM_E}{4\pi^2})^{1/3} \tag{8}$$

$$\text{(in cgs)}\ R = (\frac{86400 * 6.67x10^{-8} * 6x10^{27}}{4\pi^2})^{1/3} = 4.2x10^9 cm \tag{9}$$

4. Common Mistakes

-Geosynchronous orbit means that the satellite orbits the Earth once per day, that is, it is in a stationary position above a particular point on the Earth's surface.

-A simple ratio of periods and radii such as $\frac{P_s}{P_m} = \frac{R_s}{R_m}$ is not adequate to solve this problem.

-Many students were not careful when dealing with the units involved in the problem. Specifically, in this problem, the conversion to years and A.U. is not advised, since we are looking at orbits around the Earth and not the Sun.